

**Given :**

$$\frac{BD}{DC}=\frac{1}{2}$$

To Prove : GK $∥$ BC

Proof :

Let AF = a

AK = b

KC = c for ease.

In FBCA,

FD, BA, CG are concurrent at E.

$∴$ Bye Ceva's

$$\frac{BD}{DC}×\frac{CA}{AF}×\frac{FG}{GB}=1$$

$$⇒\frac{1}{2}×\frac{(b+c)}{a}×\frac{FG}{GB}=1$$

$⇒\frac{FG}{GB}×\frac{2a}{b+c}$ ---------------------------(1)

In $∆$ ABC,

AD,BK,CE meet at H

$$∴\frac{BD}{DC}×\frac{CK}{KA}×\frac{AE}{EB}=1$$

$$⇒\frac{1}{2}×\frac{c}{b}×\frac{AE}{EB}=1$$

$⇒$ $\frac{AE}{EB}=\frac{2b}{c}$

In $∆$ABC, FD transversal

by Menulaus,

$$\frac{CF}{FA}×\frac{AE}{EB}×\frac{BD}{DC}=1$$

$⇒$ $\frac{a+b+c}{a}×\frac{2b}{c}×\frac{1}{2}=1$

$⇒$ $b(a+b+c)$ = ac

$⇒$ $ab+b^{2}+bc=ac$

 +ac +ac

$$⇒ ab+b^{2}+bc+ac=2ac$$

$⇒$ $b\left(a+b\right)+c\left(a+b\right)=2ac$

$⇒$ (b+c)(a+b)=2ac

$$⇒\frac{2a}{b+c}=\frac{a+b}{c}$$

$⇒\frac{2a}{b+c}=\frac{FK}{KC}$

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$$⇒\frac{FG}{GB}=\frac{FK}{KC}$$

by inverse BPT, GK $∥$ BC

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